Disputes, Debt and Equity

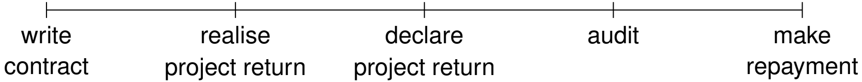
Alfred Duncan[1] Charles Nolan[2]

[1]University of Kent

[2]University of Glasgow

November 2, 2018

Timeline: Costly State Verification



Literature

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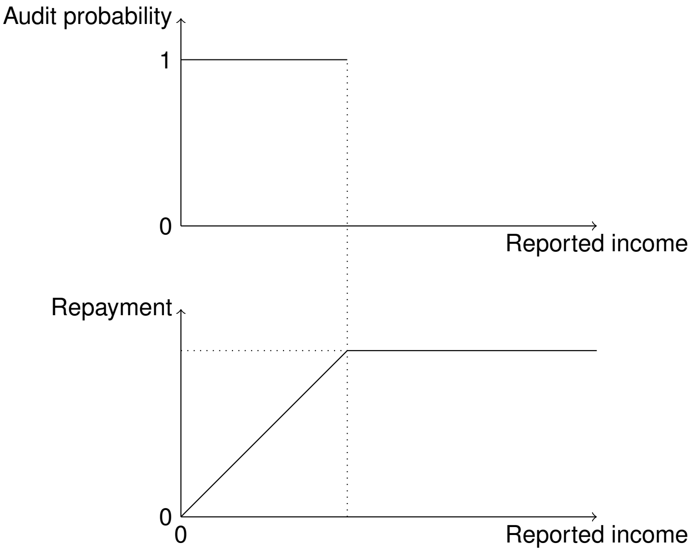
See also

- Harsanyi (1973, IJGT).
- Bolton (1986, PhD).
- Bernanke and Gertler (1989, AER).

Standard debt - Graphical definition



Notation

Nature

θ_i	Revenue shock
σ	Audit signal

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$\Delta(\theta), \Delta(\theta\|\sigma)$	Shock distributions

Contract terms, $\mathcal{C}$

$x_i(\theta, \sigma)$	Consumption ($m = m_i$)
$z_i(\sigma)$	Repayment ($m = m_i$)
q_i	Audit probability ($m = m_i$)
b	Initial transfer (amount "borrowed")

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Other variables

m	Reported revenue shock
$\mathcal{U}$	Utility, concave, unbounded above
Δ(args)	Probability measure generating function

Constructing probability measures

$$\sum_{\sigma} \Delta(\theta_i, \sigma | q) = (1 - q) \sum_{\sigma} \Delta(\theta_i) + q \sum_{\sigma} \Delta(\theta_i, \sigma)$$

Incentive compatibility

Revelation principle: Restrict our attention to contracts that elicit truth-telling.

$$m_i \in \arg\max_{m} \sum_{\sigma} \Delta(\theta_i, \sigma | \mathcal{Q}(m)) \mathcal{U}(\theta_i, m, \sigma) \quad \forall i \in \{1, 2, ..., n\}$$

Incentive compatibility

Revelation principle: Restrict our attention to contracts that elicit truth-telling.

$$\sum_{\sigma} \Delta(\theta_i, \sigma | q_i)\, \mathcal{U}(\theta_i, m_i, \sigma) \geq \sum_{\sigma} \Delta(\theta_i, \sigma | q_j)\, \mathcal{U}(\theta_i, m_j, \sigma) \geq 0 \quad \forall\, i, j < i.$$

Programme 1

$$\max_{\mathcal{C}} \sum_{i,\sigma} \Delta(\theta_i, \sigma | q_i) \, \mathcal{U}(\theta_i, m_i, \sigma)$$

Subject to

$$\sum_{\sigma,i} \Delta(\theta_i, \sigma | q_i) z_i(\sigma) - b\rho - \sum_i \Delta(\theta_i) q_i (\alpha + b)\kappa \geq 0.$$

$$\sum_{\sigma} \Delta(\theta_i, \sigma | q_i) \, \mathcal{U}(\theta_i, m_i, \sigma) - \sum_{\sigma} \Delta(\theta_i, \sigma | q_j) \, \mathcal{U}(\theta_i, m_j, \sigma) \geq 0 \quad \forall \; i, j < i,$$

plus ex post budget constraints, and $q_i \in [0, 1]$.

Proposition 1

Let $\mathcal{C}$ be a globally optimal contract. $\mathcal{C}$ satisfies the following necessary conditions.

Repayments $\mathcal{Z}(m_i, \sigma)$ satisfy

$$\frac{\mathcal{U}'(\theta_i, m_i, \sigma)}{\lambda} = \frac{1}{1 + \sum_k \mu_{i,k}} \left[1 + \sum_j \mu_{j,i} \frac{\Delta(\theta_j|\sigma)}{\Delta(\theta_i|\sigma)} \frac{\mathcal{U}'(\theta_j, m_i, \sigma)}{\lambda} \right]$$

for all (m_i, σ),

$$\lambda > 0, \quad \mu_{j,i}, \geq 0 \; \forall \; i, j, k.$$

Where λ is the Kuhn-Tucker multiplier on the Participation constraint, $\mu_{j,i}$ is the Kuhn-Tucker multiplier on the ICC with $j > i$.

Proposition 1
Let $\mathcal{C}$ be a globally optimal contract. $\mathcal{C}$ satisfies the following necessary conditions.

The initial transfer of resources to the entrepreneur b satisfies

$$\sum_{i} \Delta(\theta_i)[\theta_i - q(\theta_i)\kappa] - \rho = \sum_{i,\sigma} \Delta(\theta_i, \sigma | q_i) \left[\sum_{j} \mu_{j,i} \frac{\Delta(\theta_j|\sigma)}{\Delta(\theta_i|\sigma)} \frac{\mathcal{U}'(\theta_j, m_i, \sigma)}{\lambda} (\theta_j - \theta_i) \right],$$

for all $i \in \{1, 2, ..., n\}$, and

$$\lambda > 0, \quad \mu_{j,i}, \geq 0 \; \forall \; i, j, k.$$

Proposition 1 - Comments on Proof

- Programme 1 is non-convex.
- Slater conditions not satisfied—necessity of first order conditions is not guaranteed.
- We split the problem in two
  - Outer: Audit strategy (FOCs not necessary)
  - Inner: Repayments and amount borrowed (FOCs necessary)
- Necessity of FOCs proved using Mangasarian Fromowitz (1967) constraint qualification.

Perfect audits

Theorem 1

(Mookherjee and Png, 1989) Under perfect audits ($\Delta(\theta_i|\sigma_k) \in \{0, 1\} \; \forall \; i, k$), any optimal contract without certain immiseration following truthful reports $\left(\sum_\sigma \Delta(\theta_i, \sigma|q_i)\mathcal{U}(\theta_i, m_i, \sigma) > \underline{\mathcal{U}} \; \forall \; i\right)$ cannot include certain auditing of any report. That is, $q_i < 1 \; \forall \; i$.

Theorem 1 - proof notes

Let report *j* be audited with certainty. Perfect audits means that we can assign audit signals directly to revenue states, $\Delta(\theta_i|\sigma_i) = 1$.

The incentive constraint for agent receiving $\theta_i > \theta_j$ and considering report m_j is

$$\sum_{\sigma} \Delta(\theta_i, \sigma | q_i)\, \mathcal{U}(\theta_i, m_i, \sigma) - \sum_{\sigma} \Delta(\theta_i, \sigma_j)\, \mathcal{U}(\theta_i, m_j, \sigma_j) \geq 0 \quad \forall\, i, j < i,$$

Can always set $\mathcal{U}(\theta_i, m_j, \sigma_j)$ sufficiently low to ensure that the constraint is non-binding, without affecting the consumption of any truth-telling agent, who could never recieve audit signal σ_j.

Theorem 1 - proof notes

- We recast the proof that provided by Mookherjee and Png (1989) to demonstrate the link to the audit quality.
- We show that if $q_i = 1$ for some i, then the participation constraint could be relaxed by replacing the perfect audit technology with a synthetic, imperfect audit technology generated by combining the perfect technology with a lottery.
- This perturbation reduces audit costs and has no effect on allocations or incentive compatiblity.

Imperfect audits

Assumption 3
(Imperfect audits) The probabilities of all revenue states are positive conditional upon any audit signal, ($\Delta(\theta_i|\sigma) > 0, \ \forall \ \theta_i, \sigma$).

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Proof notes

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    - Maximal rewards result of Border and Sobel (1987)

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- Departure from maximal deterrence.
  - Departure from unobservable effort (Bolton, 1986)
- Requires risk aversion (Mookherjee and Png, 1989)
  - Maximal rewards result of Border and Sobel (1987)
- Existence of optimal leverage requires imperfect audits.

Part (a) Lower limit of consumption: Case 2. Utility is unbounded below.

Sequence of contracts with the property that $x_i^s(\theta_i, \sigma) \rightarrow 0$ for some (θ_i, σ) such that $\Delta^s(\theta_i, m_i, \sigma) > 0$. Since $\mathcal{U}(\theta_i, m_i, \sigma) \rightarrow -\infty,\ \Delta^s(\theta_i, m_i, \sigma) \rightarrow 0$.

Part (a) Lower limit of consumption: Case 2. Utility is unbounded below.

Sequence of contracts with the property that $x_i^s(\theta_i, \sigma) \to 0$ for some (θ_i, σ) such that $\Delta^s(\theta_i, m_i, \sigma) > 0$. Since $\mathcal{U}(\theta_i, m_i, \sigma) \to -\infty$, $\Delta^s(\theta_i, m_i, \sigma) \to 0$.

Let contract $\mathcal{C}^*$ be defined as follows: All contract terms in $\mathcal{C}^*$ are equal to those specified by $\mathcal{C}^s$ aside from the following:

$$x_i^*(\theta_i, \sigma) = (\alpha + b)\theta_i, \qquad z_i^*(\sigma) = 0.$$

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Satisfies the ex post budget constraints, participation constraints, increases expected utility.

Part (a) Lower limit of consumption: Case 2. Utility is unbounded below.

Consider a binding incentive compatibility constraint for an agent earning $\theta_j > \theta_i$. Under the new contract, this binding constraint is tightened by

$$\Delta^s(\theta_j, m_i, \sigma)\mathcal{U}((\alpha + b^s)(\theta_j - \theta_i) + x_i^s(\theta_i, \sigma)).$$

The utility allocation

$$\mathcal{U}((\alpha + b^s)(\theta_j - \theta_i) + x_i^s(\theta_i, \sigma)) \to \mathcal{U}((\alpha + b^s)(\theta_j - \theta_i))$$

The probability $\Delta^s(\theta_j, m_i, \sigma) \to 0$.

It follows that

$$\Delta^s(\theta_j, m_i, \sigma)\mathcal{U}((\alpha + b^s)(\theta_j - \theta_i) + x_i^s(\theta_i, \sigma)) \to 0.$$

The ICC remains satisfied after the perturbation.

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When audit costs κ are sufficiently small, optimal contracts are standard debt contracts.

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- Requires imperfect audits
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- Requires imperfect audits
  - Exogenous limited liability constraints do not generate the marginal risk sharing benefit obtained by increasing audit probability and reducing penalties.
  - Ensures that the marginal value of additional information is always positive.

Theorem 2 - Sketch of Proof

Set $\kappa = 0$

- Lemma 2: Under any audit technology there exists a standard debt contract that is optimal (not the unique optimal contract).

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- Lemma 3: If there is an audit signal σ that is informative for revenue states $\theta_i, \theta_j,\ i < j$,

$$\Delta(\theta_i|\sigma) \neq \Delta(\theta_j|\sigma),$$

  then

  - There is no optimal contract with $q_i < 1$.
  - For all $k < i$, the signal σ must be informative for either the pair k, i or k, j.
  - For all $k < i$, there is no optimal contract with $q_k < 1$.

Theorem 2 - Sketch of Proof

Set $\kappa = 0$

- Find the maximum i such that there is σ that is informative for revenue states $\theta_i, \theta_j,\ i < j$. Then optimal contracts require
  - $q_k = 1$ for all $k < i$
  - q_l for all $l > i$ can take any value (no marginal cost of audits)

Theorem 2 - Sketch of Proof

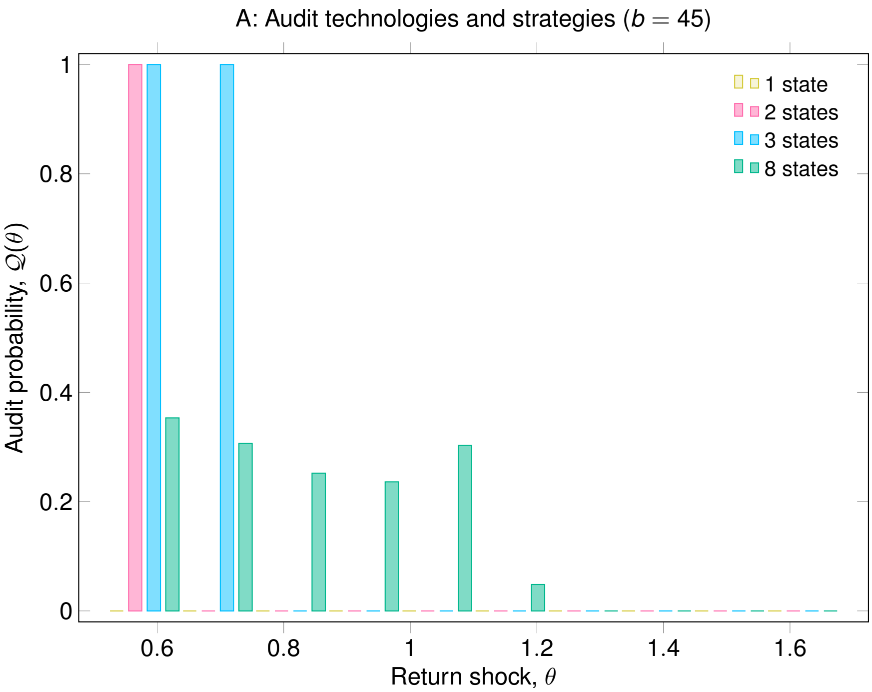
Set $\kappa > 0$

- Now we have a marginal cost of audits.
- For sufficiently low κ, optimal contracts require
  - $q_k = 1$ for all $k < i$
  - $q_l = 0$ for all $l > i$

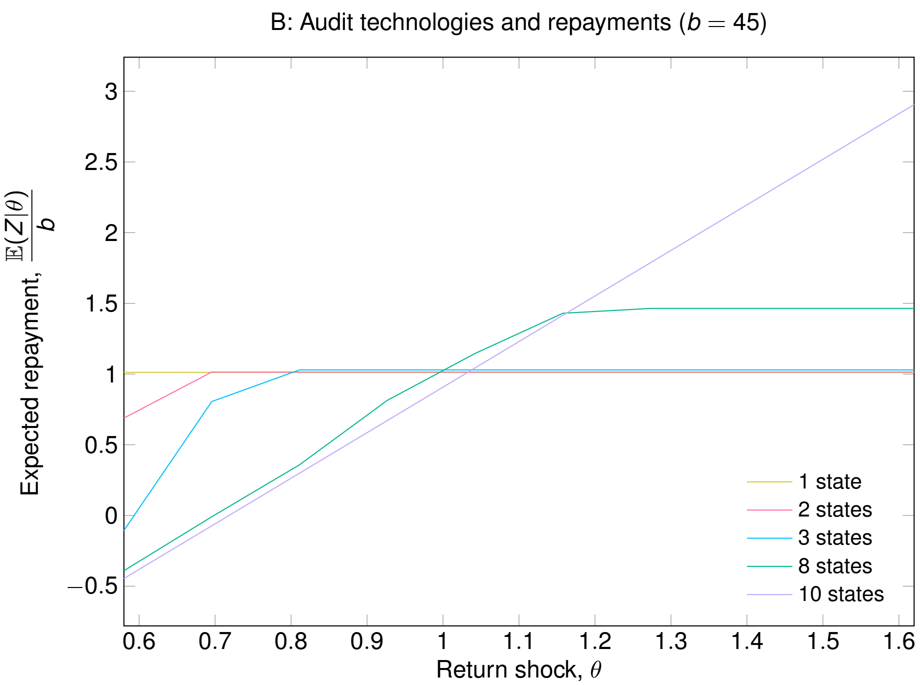
Numerical example

Table : Calibration

	Description	Data	Model
Unmatched parameters			
	Return bounds, $\{\theta_1, \theta_{10}\}$		$\{0.58, 1.62\}$
	Distribution parameter, p		0.435
	Number of signals, $\lvert\Sigma\rvert$		3
Matched parameters			
	Opportunity cost, ρ	1.01	1.012
	Audit costs, κ	(0.01,0.06)	0.012
Matched predicted moments			
	Borrowing, b^*	44.9	45.0
	Probability of default	0.040	0.047
	Credit spread	0.039	0.017

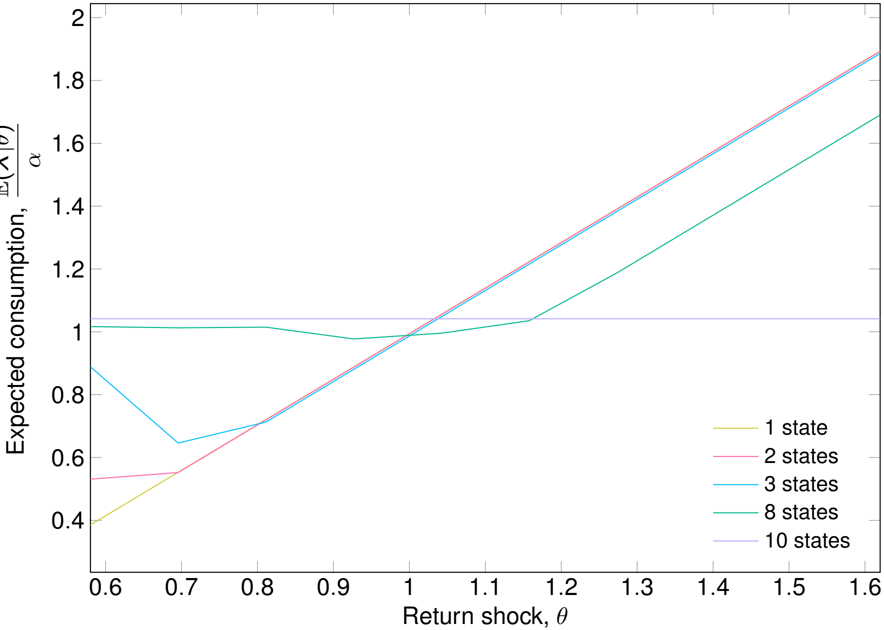


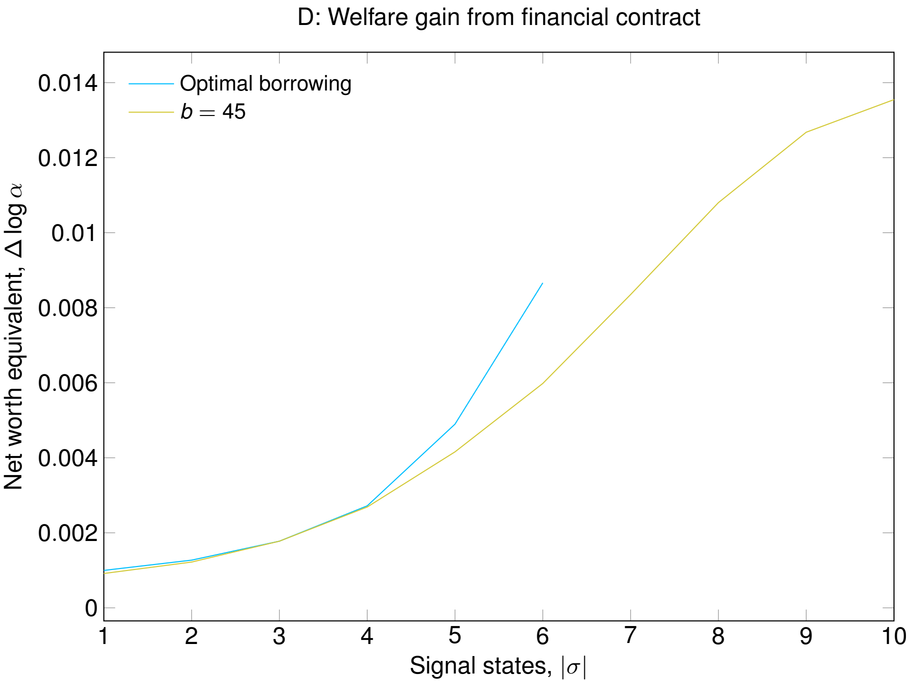
A: Audit technologies and strategies ($b = 45$)
Audit probability, $\mathcal{Q}(\theta)$
Return shock, θ
1 state
2 states
3 states
8 states
0
0.2
0.4
0.6
0.8
1
0.6
0.8
1
1.2
1.4
1.6



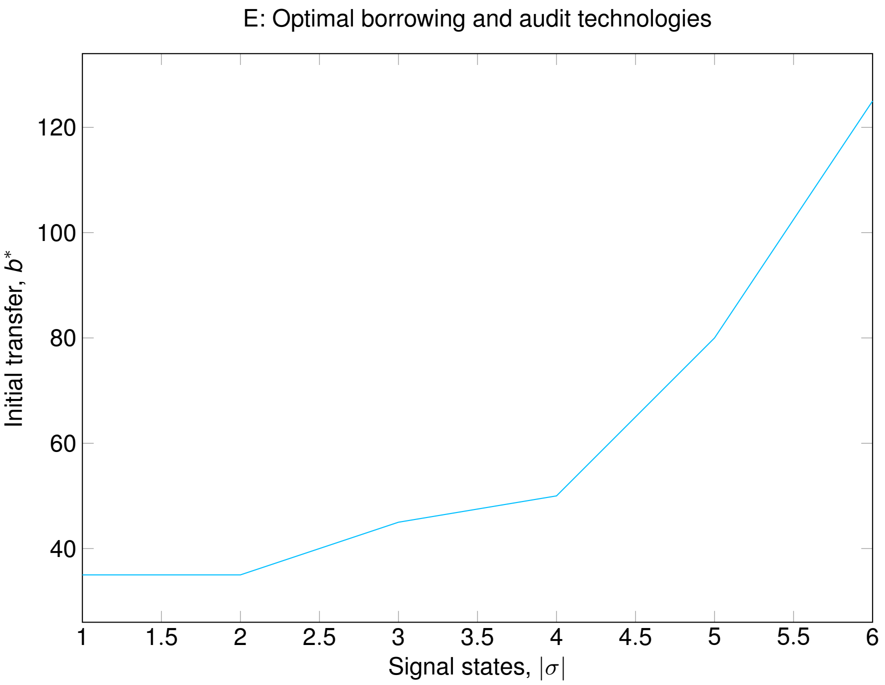
B: Audit technologies and repayments ($b = 45$)
Expected repayment, $\frac{\mathbb{E}(Z|\theta)}{b}$
Return shock, θ
3
2.5
2
1.5
1
0.5
0
−0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
1 state
2 states
3 states
8 states
10 states

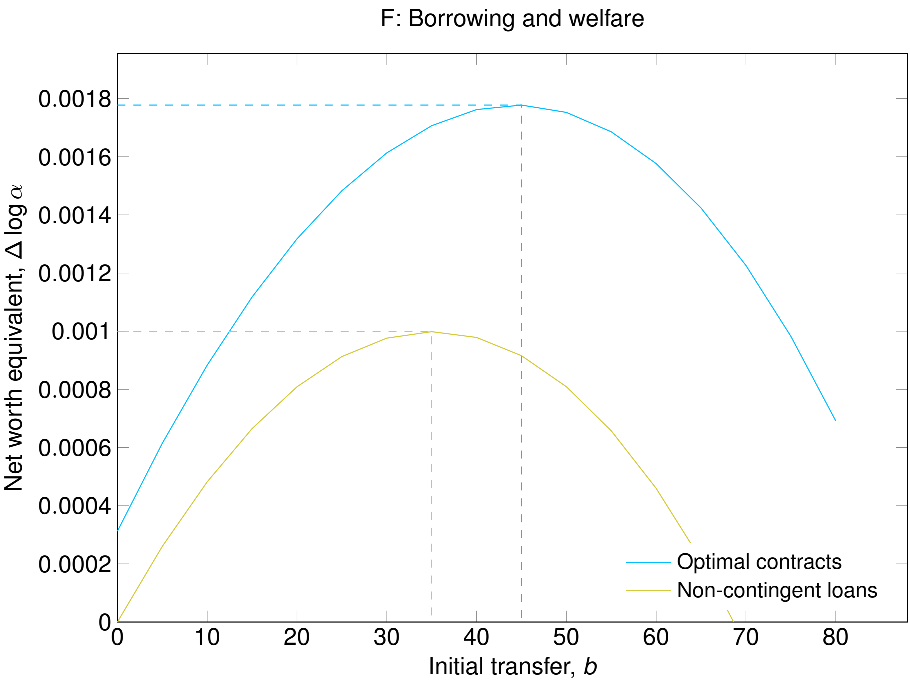
C: Audit technologies and consumption ($b = 45$)





D: Welfare gain from financial contract
Optimal borrowing
$b = 45$
Net worth equivalent, $\Delta \log \alpha$
Signal states, $|\sigma|$
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
1
2
3
4
5
6
7
8
9
10


E: Optimal borrowing and audit technologies
Initial transfer, b^*
Signal states, $|\sigma|$
120
100
80
60
40
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6


F: Borrowing and welfare
Net worth equivalent, $\Delta \log \alpha$
Initial transfer, b
Optimal contracts
Non-contingent loans
0.0018
0.0016
0.0014
0.0012
0.001
0.0008
0.0006
0.0004
0.0002
0
0
10
20
30
40
50
60
70
80

Closed form solutions

Closed form solutions

In a two state, two signal version of the model, we generate closed form solutions under CRRA utility.

Proposition 7
When the likelihood of Type-I and Type-II errors are positive and zero respectively

$$\Delta(\theta_1, \sigma_2) > 0 \; \forall \sigma, \; \exists \sigma' \; \Delta(\theta_2, \sigma_1) = 0$$

and preferences exhibit constant relative risk aversion $U(x) = x^{1-\gamma}/(1-\gamma)$, leverage, consumption allocations and shadow prices of standard debt contracts and non-contingent debt contracts can be represented by closed-form expressions in terms of exogenous parameters.

Example of Proposition 7 with log utility

$$\frac{\alpha + b}{\alpha} = \frac{\rho}{\theta_2 - \theta_1} \frac{\zeta}{1 - \zeta} \left(\frac{\Delta(\theta_2) + \Delta(\theta_1, \sigma_2)}{\Delta(\theta_2)\zeta + \Delta(\theta_1, \sigma_2)} \right)$$

where

$$\zeta = \frac{\mathbb{E}[\theta] - \rho - \Delta(\theta_1)\kappa}{\Delta(\theta_2)(\theta_2 - \theta_1)} \qquad \in (0, 1)$$

$$\Delta(\theta_2, \sigma_2) > 0$$

$\Delta(\theta_1)$ is the probability of default.

$$\zeta = \sqrt{\frac{\Delta(\theta_2)}{\Delta(\theta_1)}} \text{[Sharpe Ratio]}$$