Private Information and Business Cycle Risk Sharing

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Research questions

- How do markets allocate business cycle risk across agents with different levels of wealth?
- Is the market allocation (constrained) efficient?
- How does the constrained efficient allocation differ from the market allocation?

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Policy implications

- Design of welfare programmes and their response to business cycle fluctuations.
- Design of macroprudential policy (broadly defined as intervention in the allocation of business cycle risks).

Preview

We study an environment where private information about individual specific risks prevents income insurance markets from opening.

- How do markets allocate business cycle risk across agents with different levels of wealth?
  - Under sequential trade, markets equate agents' consumption MRS across business cycle outcomes.
- Is the market allocation (constrained) efficient?
  - Not under sequential trade.
- How does the constrained efficient allocation differ from the market allocation?
  - The constrained efficient allocation reduces the exposure of low wealth individuals to business cycle fluctuations.

Intuition

- Business cycle risk (think exposure to the stock market) is a complement to misreporting low income.
- Restricting access to business cycle risk for low reporting agents relaxes incentive constraints relating to truthful reporting of individual specific risks.
- This permits a greater sharing of individual specific risks.

Don't buy anything. Don't Get anything. What's the matter with you? ... Because your going to get us all pinched. - Goodfellas (1990)

Literature

- Cole and Kocherlakota (REStud, 2001), Phelan (REStud, 1994), Green and Oh (QR, 1991).
  - Constrained efficient allocations decentralisable when storage is hidden and no aggregate risk.

Literature

- Duncan and Nolan (2017), Farhi and Werning (ECMA, 2016).
  - Market allocation of aggregate risk constrained inefficient in the presence of financial frictions, New Keynesian markups.

Literature

- Fisher Johnson and Smeeding (AER, 2013), Perri and Steinberg (2012), Meyer and Sullivan (AER, 2013).
    - Low wealth households less sensitive in log consumption terms to business cycles.

Literature

- Mitman and Rabinovich (JME 2015), Williams and Li (2015), Landais, Michaillat and Saez (2013).
  - Optimal employment insurance and the business cycle—focused on vacancy costs.

Model

- Two period endowment economy model.
- Idiosyncratic risk in the first period (private information).
- Aggregate risk in the second period.
- Hidden durable good.
- Decreasing Absolute Risk Aversion (DARA) preferences
  - Following an increase in wealth, agents increase their holdings of risky assets.
  - DARA: $A'(c) < 0$ where $A(c) = \frac{-u''(c)}{u'(c)}$.
  - CRRA $\subset$ DARA
  - DARA $\rightarrow U'''(c) > 0$.

Model - no business cycle risk

$$\max_{\mathbf{c},x} \pi_l U(c_{1l}) + \pi_h U(c_{1h}) + \beta[\pi_l U(c_{2l}) + \pi_h U(c_{2h})] \tag{1}$$

Resource constraints (Lagrange multipliers λ_1, λ_2):

$$\pi_l y_l + \pi_h y_h \geq \pi_l c_{1l} + \pi_h c_{1h} + x \tag{2}$$

$$z + Rx \geq \pi_l c_{2l} + \pi_h c_{2h} \tag{3}$$

Perfect information

FONCs

$$\begin{aligned} c_{1i}: \quad & \lambda_1 = U'(c_{1l}) = U'(c_{1h}) \\ c_{2i}: \quad & \lambda_2/\beta = U'(c_{2l}) = U'(c_{2h}) \\ x: \quad & \lambda_1 = R\lambda_2 \end{aligned}$$

First best efficient allocations

$$c_{1l} = c_{1h} = c_{2l} = c_{2h} = \frac{1}{1+\beta}\left[\pi_l y_l + \pi_h y_h + \beta z\right].$$

Private information - the planner's problem

$$\max_{\mathbf{c},x} \pi_l U(c_{1l}) + \pi_h U(c_{1h}) + \beta[\pi_l U(c_{2l}) + \pi_h U(c_{2h})] \tag{4}$$

Resource constraints (Lagrange multipliers λ_1, λ_2):

$$\pi_l y_l + \pi_h y_h \geq \pi_l c_{1l} + \pi_h c_{1h} + x \tag{5}$$

$$z + Rx \geq \pi_l c_{2l} + \pi_h c_{2h} \tag{6}$$

Truth telling constraint (μ)

$$U(c_{1h}) + \beta U(c_{2h}) = V(c_{1l} + y_h - y_l, c_{2l}). \tag{7}$$

Private information - the value of misreporting

Truth telling constraint (μ)

$$U(c_{1h}) + \beta U(c_{2h}) = V(c_{1l} + y_h - y_l, c_{2l}). \tag{8}$$

$$V(c_{1l} + y_h - y_l, c_{2l}) = \max_{\hat{\mathbf{c}}, x} U(\hat{c}_1) + \beta U(\hat{c}_2)$$

subject to the resource constraints

$$c_{1l} + y_h - y_l \geq \hat{c}_1 + \hat{x},$$

$$R\hat{x} + c_{2l} \geq \hat{c}_2.$$

Allocations:

$$\hat{c}_1 = \hat{c}_2 = \frac{1}{1+\beta}\left[c_{1l} + y_h - y_l + \beta c_{2l}\right]$$

$$V(c_{1l} + y_h - y_l, c_{2l}) = (1+\beta)U\left(\frac{1}{1+\beta}\left[c_{1l} + y_h - y_l + \beta c_{2l}\right]\right) \tag{9}$$

Private information - the planner's solution

FONCs

$$
\begin{aligned}
c_{1l}: &\quad \pi_l \lambda_1 = \pi_l U'(c_{1l}) - \mu U' \left(\frac{1}{1+\beta} \left[c_{1l} + y_h - y_l + \beta c_{2l} \right] \right) \\
c_{2l}: &\quad \pi_l \lambda_2 = \pi_l \beta U'(c_{2l}) - \mu \beta U' \left(\frac{1}{1+\beta} \left[c_{1l} + y_h - y_l + \beta c_{2l} \right] \right) \\
c_{1h}: &\quad \pi_h \lambda_1 = \pi_h U'(c_{1h}) + \mu U'(c_{1h}) \\
c_{2h}: &\quad \pi_h \lambda_2 = \pi_h \beta U'(c_{2h}) + \mu \beta U'(c_{2h}) \\
x: &\quad \lambda_1 = R \lambda_2
\end{aligned}
$$

The solution to this problem is

$$
c_{1l} = c_{2l} = \frac{1}{1+\beta}[y_l + \beta z], \qquad c_{1h} = c_{2h} = \frac{1}{1+\beta}[y_h + \beta z]. \tag{10}
$$

Decentralisability (Cole and Kocherlakota)

Proposition

When aggregate income is constant, the constrained efficient allocations under private information with hidden storage can be implemented with decentralised trade in non-contingent one period debt contracts.

Aggregate risk

Aggregate risk - the planner's problem

$$\max_{\mathbf{c},x} \ \pi_l U(c_{1l}) + \pi_h U(c_{1h}) + \beta\, \mathbb{E}_z \left[\pi_1 U(c_{2l}(z)) + \pi_h U(c_{2h}(z))\right],$$

subject to the budget constraints,

$$\pi_l y_l + \pi_h y_h \geq \pi_l c_{1l} + \pi_h c_{1h} + x, \qquad (\lambda_1)$$

$$Rx + z \geq \pi_l c_{2l}(z) + \pi_h c_{2h}(z) \qquad z \in \{z_L, z_H\}, \qquad (\lambda_2(z))$$

and the incentive constraint,

$$U(c_{1h}) + \beta \mathbb{E} U(c_{2h}(z)) \geq V(c_{1l} + y_h - y_l, c_{2l}(z)). \qquad (\mu)$$

The value of misreporting

Lemma

Under any allocation where low endowment agents' ex ante intertemporal marginal rates of substitution are equated to the gross return on the savings technology $\left(\frac{U'(c_{1l})}{\beta \mathbb{E}_z U'(c_{2l}(z))} = R\right)$*, it follows that*

(a) *any misreporting agent saves a strictly positive amount of their first period wealth,* $\hat{x} > 0$*.*

(b) *Misreporting agents' ex ante intertemporal marginal rate of substitution is equated to the gross return on savings,*

$$\frac{U'(\hat{c}_1)}{\beta \mathbb{E}_z U'(\hat{c}_2(z))} = R. \tag{11}$$

(c) *Under DARA preferences, the across state marginal rate of substitution of misreporting agents is strictly greater than that of truth-telling low endowment agents,*

$$\frac{U(\hat{c}_2(z_H))}{U(\hat{c}_2(z_L))} > \frac{U(c_{2l}(z_H))}{U(c_{2l}(z_L))}. \tag{12}$$

Protect low wealth households from business cycles

Proposition

When agents' preferences exhibit DARA, then under the constrained efficient allocations, the period 2 across-state consumption marginal rate of substitution of low wealth agents is greater than that of high endowment agents:

$$1 > \frac{U'(c_{2l}(z_H))}{U'(c_{2l}(z_L))} > \frac{U'(c_{2h}(z_H))}{U'(c_{2h}(z_L))}. \tag{13}$$

CE with non-contingent debt

The agent's problem can be written as follows:

$$\max_{\mathbf{c}_i, x_i, b_i} U(c_{1i}) + \beta U(c_{2i})$$

subject to the resource constraints

$$y_i \geq c_{1i} + x_i + Qb_i, \quad (\lambda_{1i})$$

$$Rx_i + b_i + z \geq c_{2i}(z). \quad (\lambda_{2i}(z))$$

In symmetric equilibrium, the total supply of one period bonds must be equal to zero:

$$\pi_l b_l + \pi_h b_h = 0.$$

The first order necessary conditions are

$$\begin{aligned} c_{1i}: &\quad \lambda_{1i} = U'(c_{1i}) \\ c_{2i}: &\quad \lambda_{2i} = \beta U'(c_{2i}(z)) \\ x: &\quad \lambda_{1i} = R\mathbb{E}\lambda_{2i}(z) \\ b: &\quad Q\lambda_{1i} = \mathbb{E}\lambda_{2i}(z) \end{aligned}$$

CE with non-contingent debt

Proposition

With business cycle risk present and DARA preferences, and with trade in non-contingent debt contracts only, the period 2 across-state consumption marginal rate of substitution of low wealth agents is less than that of high endowment agents:

$$\frac{U'(c_{2l}(z_H))}{U'(c_{2l}(z_L))} < \frac{U'(c_{2h}(z_H))}{U'(c_{2h}(z_L))}. \tag{14}$$

CE with state-contingent debt

The agent's problem can be written as follows:

$$\max_{\mathbf{c}_i, x_i, b_i} U(c_{1i}) + \beta U(c_{2i})$$

subject to the resource constraints

$$y_i \geq c_{1i} + x_i + Q(z)b_i(z), \quad (\lambda_{1i})$$

$$Rx_i + b_i(z) + z \geq c_{2i}(z). \quad (\lambda_{2i}(z))$$

In symmetric equilibrium, the total supply of bonds contingent on state z must be equal to zero:

$$\pi_l b_l(z) + \pi_h b_h(z) = 0 \qquad \forall z.$$

The first order necessary conditions are

$$\begin{aligned} c_{1i}: &\quad \lambda_{1i} = U'(c_{1i}) \\ c_{2i}: &\quad \lambda_{2i} = \beta U'(c_{2i}(z)) \\ x: &\quad \lambda_{1i} = R\mathbb{E}\lambda_{2i}(z) \\ b(z): &\quad Q(z)\lambda_{1i} = \lambda_{2i}(z) \qquad \forall z \end{aligned}$$

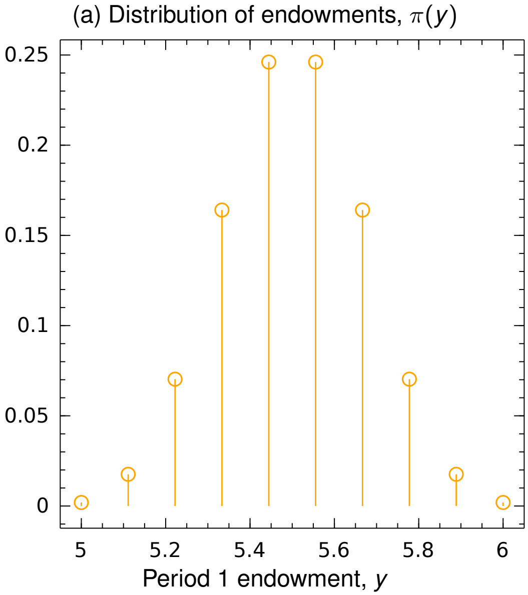
CE with non-contingent debt

Proposition

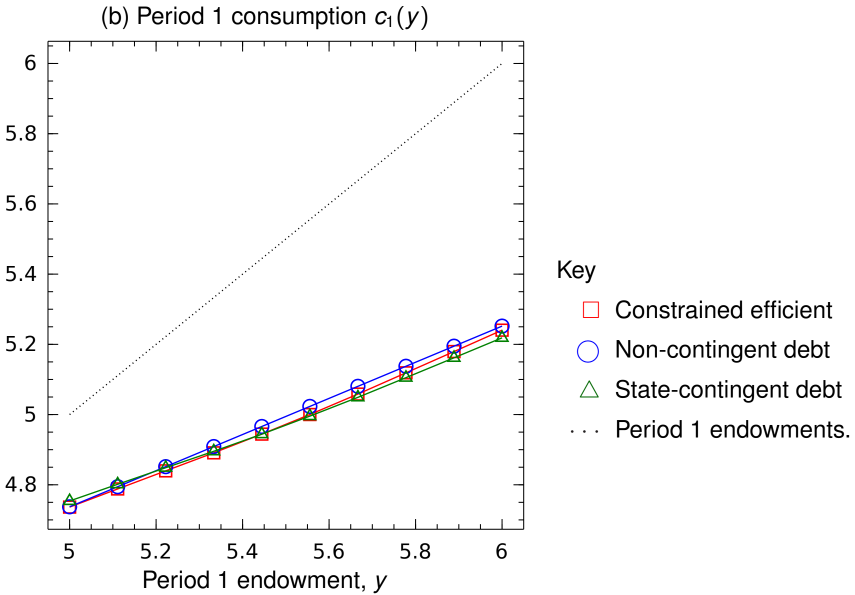
With trade in both non-contingent debt and one period securities contingent on the aggregate shock z, the period 2 across-state consumption marginal rate of substitution of low wealth agents is equal than that of high endowment agents:

$$\frac{U'(c_{2l}(z_H))}{U'(c_{2l}(z_L))} = \frac{U'(c_{2h}(z_H))}{U'(c_{2h}(z_L))}. \tag{15}$$

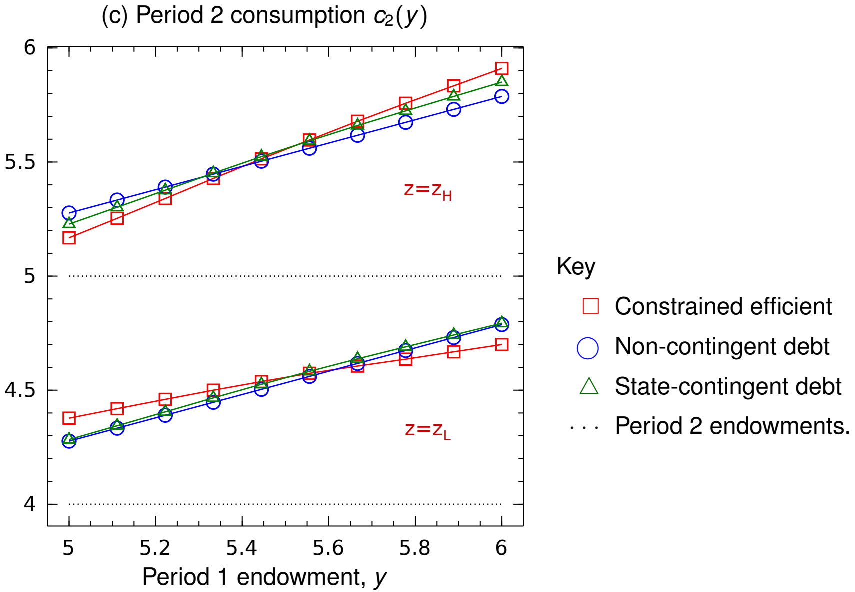
Numerical example



Numerical example

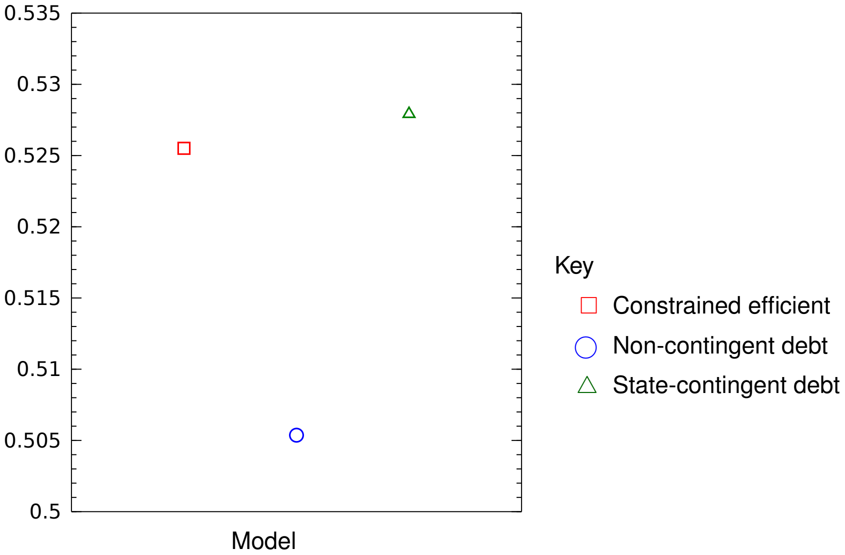


Numerical example



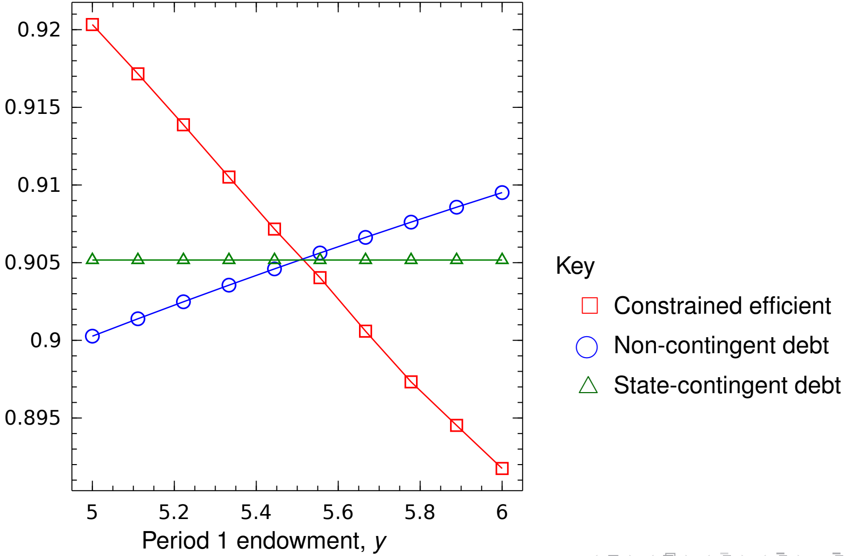
Numerical example

(d) Period 1 savings, $\sum_y \pi(y)(y - c_1(y))$



Numerical example

(e) Period 2 across-state MRS, $\frac{U'(c_2(y, z_H))}{U'(c_2(y, z_L))}$



Review

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